The mouse was a German mathematics professor, David Hilbert, who accomplished many feats of creativity and order along the Stream of Mathematics.

On sunny mornings, Hilbert the mouse would sit amidst patterns of twigs and gleaming pebbles and propound, to the gathered assembly of admiring creatures, 'We must use logic to prove the consistency of mathematics, since—*squeak!*—I've already shown...that we can map all of mathematics onto arithmetic.'

And his hearers, bobbing enthusiastically, would scamper off, crying, 'Yes! To work! To work!'

The frog was the Dutch mathematician, Luitzen Egbertus Jan Brouwer. One blue-strewn day, discarding his tadpole tail with a grunt, Brouwer the Frog swam to a lily pad, assumed the lotus position, and delivered, to the shocked passersby on the bank, his doctoral thesis:

'The Lord helps those who help themselves! Brek-kek-kek-kek! Hilbert the Mouse and Russell the Rabbit and the other logicians and mathematicians think they can cop out in their proofs. Brek! When it's too hard to prove a statement *directly*, they assume the *contradictory* of the statement, then seek *a contradictory of the contradiction. Broax-broax!* They suppose that the Good Lord, like a naive, orderly German professor, or an eccentric English aristocrat, has providently bundled all possible into *neat pairs of contradictories.* And, when any sagacious thinker finds an earthly copy of one of a given pair of statements to be defective, he can forward it to heaven in sacrificial smoke, and the Lord sends back its *negation* by return zephyr. Brek-kek-kek! Such nonsense! As every sensible frog knows, the Lord is an industrious ruler, who loves a constructive worker with crafty hands. If you wish a proof of a statement, you must *construct* it, *directly*, of your own initiative, and then the Lord will approve. Henceforth, I shall accept nothing as *rigorously* established in mathematics in absence of its *construction*. We may assume nothing more, except, as Kronecker the Shrew noted, *the intuition of the natural numbers*, passing endlessly by, like the ripples of this cool, sweet stream.' ...

When the creatures round about had recovered their wits, they scampered off to tell Hilbert the Mouse of this new heresy. A grasshopper vaulted after them, spitting tobacco juice and declaring: 'There's gonna be a fight! Sputt! Betcha the Mouse and the Frog will get into a scrap as soon as they meet! Sputt!'

And fight they did!—verbally, in loud angry tones that reverberated through the Academic Grove, beyond the Stream of Mathematics.

On one occasion, Hilbert squeaked to the assembly: There have been two great *crises* in *the Foundations of Mathematics*. First, when the Pythagoreans discovered *the irrationality of the square root of two* ... Second, when the philosophical Bishop, Berkeley the Bear, pointed out *the contradiction of dividing by zero* ... But Cantor the Goat, by his *General Set Theory*, has shown us how to resolve these problems, and to dispel crisis from the Foundations of Mathematics! And now we shall go forward to—.'

'Don't listen to that quietist propaganda!' boomed a voice from the rushes. 'Cantor's set theory attacked those problems only to create *a third crisis in the Foundations of Mathematics!* Hilbert is quite familiar with the paradoxes of set theory!'

'But they've been taken care of!' squeaked Hilbert, in shrill protest. 'And mathematics is the richer for Cantor's General Set Theory, and for *the chain of infinite aleph numbers* he derived therein. 'No one,' he snapped, whiskers twiddling, eyes blazing, 'no one shall drive us from the paradise Cantor created for us!'

'Broax! and nonsense! You prate constantly about building a neat little pen of axioms to protect the innocent sheep of your doctrinal theses from the wolves of contradiction. But you build by General Set Theory. And as Poincaré the Otter so aptly asked: *What good is it to build a pen for your sheep if you corral the wolves of contradiction inside with them?'*

At another time Brouwer the Frog sat on a lily pad and dispensed the new reformist gospel to a fascinated but uneasy crowd of creatures on the banks of the Stream of Mathematics.

'Hilbert defends the use—in order to prove any mathematical theorem— of the so-called rule of logic ... *Either a statement or its negation is true with no third possibility.* But logic is not a set of commandments handed down from heaven—or a rock you may skip from one side to another of the Stream of Mathematics. No! Logic is a set of (possibly) useful rules of thought or argument, abstracted from our experience with mathematics— principally from number theory and arithmetic. *As pointed out in my doctoral thesis, we are allowed to abstract a limited form of this [rule of logic] from our experience with* finite collections, *since we know that, should we doubt the conclusion derived by means of this, the conclusions can always be checked out by* finite sequence of trials! *But we have no such experienced assurance of the safety of this procedure with* infinite collections! *So we cannot ... devise an existence (or* reductio ad absurdum) proof *to posit properties of infinite collections—of which we know so little!'*

But Hilbert the Mouse was now jumping up and down on the bank in anguished distress. 'To deny ... *the law of the excluded middle,* to the mathematician

is like taking the telescope from the astronomer, or the microscope from the biologist! Don't listen to Brouwer! You've seen what a shambles there would remain of mathematics if you try to follow his *constructivist methods!*'

Croaked the Frog: 'It wouldn't be a shambles! But it also wouldn't be the Cloud-Cuckoo-Land you create with your *Axiom of Choice* ... Do you know what delusions you can arrive at by means of the Axiom of Choice?'

'No,' they cried. 'What?'

'Behold! By appealing to the Axiom of Choice, I can claim that I can dissect the *very moon* into but *five parts*, then put it back together and pop it into my mouth—like this fly! Gulp! ... This axiom assures us that every collection—be it finite or infinite ... is as countable as the ripples of this cool, sweet stream!

'Shazammm!' gasped the creatures. 'And how do we do that?'

'Hump! Don't ask Hilbert the Mouse, or Russell the Rabbit, or Zorn the Muskrat, for they don't know how ... They claim that the Lord is their *Thesis Adviser* and has neatly filed away all finite and infinite collections—into file drawers. Whenever an example, or counterexample, is required in a proof, they've only to appeal to the Lord, their Thesis Adviser, and He will beneficently retrieve it from a neat file of *ordered* manila folders and dispatch it to earth—in the latest mathematical journal.'

'Shazamm!' gasped the creatures.

'Shazamm, nothing! Hoakum!' At this moment, Brouwer filled a great bubble with air in his throat, and boomed it forth in angry explosion.

'If a criminal, in committing his crime, manages to destroy the only evidence that could be used to convict him—must I, therefore, accept the verdict that he's innocent? No! Either he can be proven *guilty as charged*, or *innocent of charge*, or we rest with the verdict of *unproven by existing evidence*. Similarly, in mathematics, a thesis can be proven or disproven *constructively, or we must declare it unproven.* Brek-kek-kek!'

Other proponents of differing schools of thought would quarrel for hours on the banks of the Stream of Mathematics ...

Weyl the Badger, long considered a disciple of Hilbert the Mouse, was so persuaded by the arguments of Brouwer the Frog that he ventured the following pessimistic opinion: 'We must learn a new modesty. We have stormed the heavens but we have succeeded only in building fog upon fog, a mist which will not support anybody who earnestly desires to stand upon it.' ...

Eventually, the different facets of the controversy factioned the creatures of the region into many little bands of quarrelling doctrinaires: Formalists, Logicists, Intuitionists, Neo-intuitionists, Finitists, etc. It just wasn't possible to keep up with the issues and actions percolating out of so many storm centers. So, the Grasshopper decided it was time to bring the controversy to focus by staging a great *debate* between the two principal (and original) adversaries, Hilbert the Mouse and Brouwer the Frog.

On the morning of the great event, all the creatures who lived along the Stream of Mathematics, and many from the neighboring Forest of Logic, were crowded under the great oak tree, in the Academic Grove, to hear the opening arguments.

But the discussion was chaired by the Ant, who announced that his neighbor, the grasshopper, unexpectedly called away, had asked him to substitute. So they began.

Within an hour, the fervour of the two speakers, and the intellectual passion surging through the crowd, set off an angry antiphonal exchange between the two proponents of totally different philosophies of mathematics.

Suddenly, into the midst of the shouting, there sprang the giddy Grasshopper, ablaze with excitement: 'Wait! Hold it! This is even better than a debate! Gödel the Fox has achieved a completely unexpected result: He has proven that *you cannot prove the consistency of arithmetic!*

'What?' exclaimed a stunned Hilbert the Mouse.

'Oh, did he really?' boomed a doubtful Brouwer the Frog.

'Yes, he did, Brouwer. Indeed, indeed, Gödel the Fox used methods you cannot quarrel with. You see, the tools of his proof are constructed according to a model he abstracted from the structure of the natural numbers, your paradigm for all of mathematics. And—Hilbert, my friend—Gödel proved that any *axiomatic system,* such as you favor (and use) must *either fail to capture all essentials of arithmetic, or else it must already have penned into it at least one of the wolves of contradiction!*'

The pervasive silence that smothered the assembly under the oak tree was broken by the voice of the practical Ant: 'Then—the debate is over. So, let us get back to our labors.' ...

And in the Meadow, led by a (classical mathematical) shepherd, the sheep bleated as they ran:

The Lord is my thesis adviser; I shall not err.
He arranges for me to be published in the
respectable journals; he teaches me how
to use the *reductio* argument.
He enshores my validity; he leads me by
the classical logic, for the truth's sake.
Yea, though I walk through the valley of
the existence proofs, I will fear no contradiction;
for he edits my work.
The Axiom of Choice and Zorn's Lemma, they comfort me.

He invites a colloquium on classical analysis,
for my participation, in the absence of the constructivists;
He frequently and approvingly abstracts me
in *Mathematical Reviews;*
my reputation flourishes internationally.
Surely, honors and grants shall follow me
all the days of my career,
And I shall rise in the ranks of the Department,
to Emeritus.

Amen.